LETTER TO CHARLES LUCY

(sheet 1 of 19 sheets)

844 N. Ave 65
Los Angeles, CA 90042
Phone (213) 256-2624
April 8, 1992

Dear Charles,

Thank you for the utterly delightful tape. The comparisons with 12-scale are musically valid and interesting, and reveal your musical background.

The interval $1/\pi$ (.318309886) may be factored into an otherwise mundane tonal formula in a spectrum of intriguing and poly-culturally significant ways. I am absolutely certain ~~than~~ that these will be far more interesting to you than the entrenched 12-scale. I will comment briefly on them.

Item 1: $(1/\pi+3)/8 = .414788735$ Octave. This Fourth superimposed 8 times gives 3 octaves plus the major Third $(1/\pi)$. It runs parallel to Jami's 17-tone tuning of 13th century Persia, and to the subsequent 22-tone scale of northern India. It is likewise analogous to "Helmholtzian" tuning. The Seventh harmonic is remarkably good in this π tuning and surpasses that of Helmholtzian. Nested 2-interval patterns occur at (1), 2, 3, 5, 7, 12, 17, 29, 41, 53, 94, 135, 176, 311, 487, 798, 1109, and 1420 tones. 1420 is an auspicious break point in the 1/4 zig-zag pattern; It factors to $2^2 \cdot 5 \cdot 71$.

Item 2: $(1/\pi+5)/9 = .59092332$ Octave. This Fifth superimposed 9 times gives 5 Octaves and the major third $(1/\pi)$. This runs parallel to the classic Indian writings wherein the Fifth has 13 s'rutis and the major third 7 s'rutis. (and the 8ve 22 srutis) (9 × 13 s'rutis equals 5 × 22 srutis + 7 s'rutis). ~~13~~/22 (2×11) is a prominent breakpoint, as is ~~1888~~/3195 $(3^2 \cdot 5 \cdot 71)$ much farther down the zig-zig pattern. It is interesting how closely the this tuning comes to 22 equal, 22 Fifths exceeding 13 Octaves by .0003 Octave.

(22 × .59092332 = 13.00031306). The difference between $1/\pi$ (.318309886) and $7/22$ (.318181818) is .000128068 Octave. The 22-scale is very effectively a cycle of $1/\pi$ Thirds.

Item 3: $(1/\pi + 2)/4 = .579577471$. This Fifth superimposed 4 times gives 2 Octaves and the major third ($1/\pi$). This is John Harrisons tuning. [Were "$\log 2(5/4)$" substituted for "$1/\pi$" we would have the recipe for quarter comma meantone $(\log_2(5/4) + 2)\,4 = .580482023$.] This is a western European tuning, and suitible to western European music. Nested 2-interval-patterns (MOS) occur at (1), 2, 3, 5, 7, 12, 19, 31, 50, 69, 88, 157, 245, 333, 421, 754 1087, at 1420. At 1420 ($2^2 \cdot 5 \cdot 71$) there is a very major break, as also noted in item 1. Something odd is going on here.

Item 4: $(1/\pi + 1)/5 = .263661977$. This minor Third superimposed 5 times gives 1 Octave and the major Third ($1/\pi$). This tuning is aligned to the Hanson tuning where 6 minor Thirds add up to an Octave and the Fifth. While quite unorthodox, this interesting tonal association is worth exploring. Nested 2-interval patterns occur at (1), 2, 3, 4, 7, 11, 15, 19, 34, 53, 72, 91, 110, 201, 311, 421, 732, 1043, and at 1775 ($5^2 \cdot 71$) at which point a major break occurs.

Item 5: $1/\pi = .318309886$. This major Third superimposed 5 times gives 1 Octave plus a Fifth at 1.591549431. (See figure 1.). While this is an extremely exotic tonal association it is harmonically economical and melodically very beautiful. And it is after all a continuous chain of $1/\pi$ projected around the Octave circle. This should reveal some of the properties of this interval.

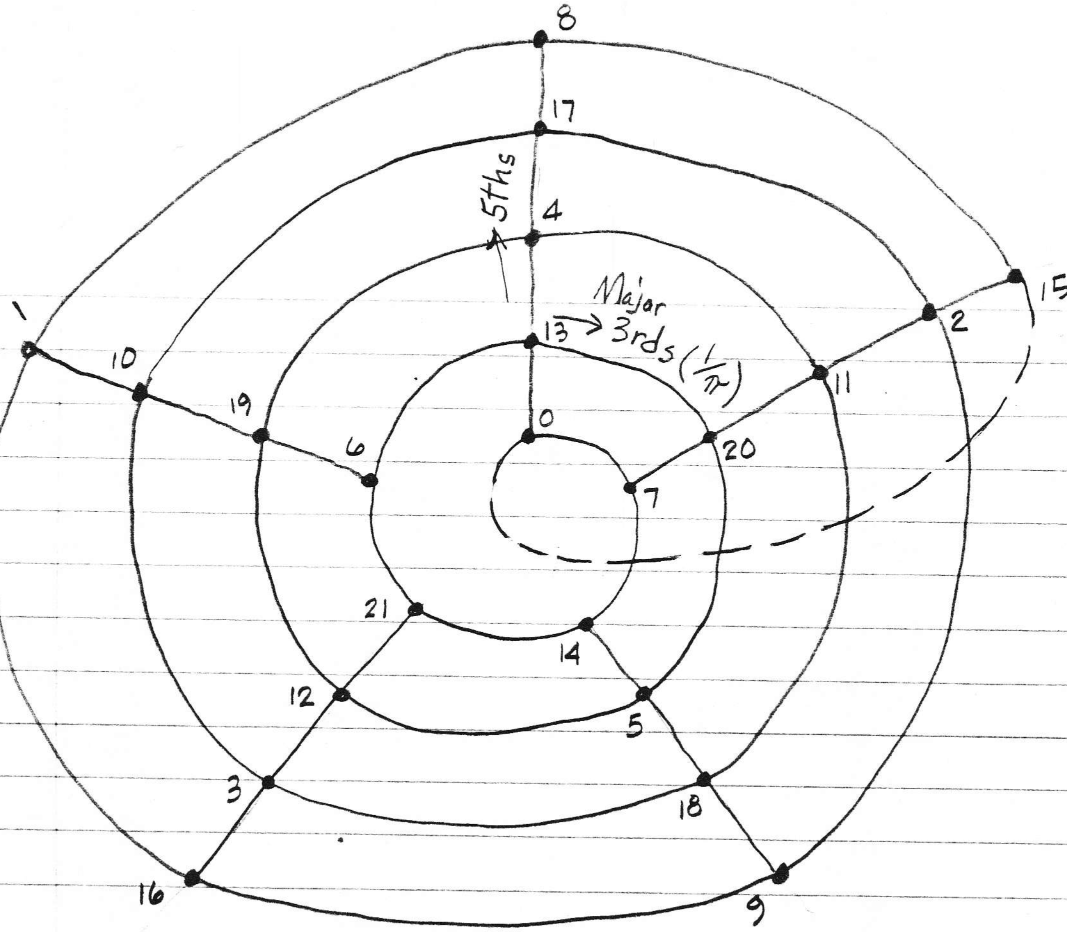


Figure 1.

– and indeed it does. The zig-zag pattern is extremely unusual, moving in bold slashes across the page. One would like to look at this pattern with quite a few more decimal places in the calculator. The nested 2-interval patterns are at (1), 2, 3, 4, 7, 10, 13, 16, 19, 22, 25, 47, 69, 91, 113, 135, 157, 179, 201, 223, 245, 267, 289, 311, 333, 355 -- with breakpoints at 3, 22 (2×11), and 355 (5×71). This explains why we're getting breakpoints in the other scales at multiples of 22 and 355. But does it explain whats going on?

Item 6: $(1/\pi + 1)/3 = .439436628$ Octave. This Large Fourth superimposed 3 times gives the Octave and a major Third ($1/\pi$). This falls within a Pelog tolerance, and altho somewhat rare, I have seen Pelog tuning tables where the "closing" Fourth of a 7-tone 2-interval pattern is very much like the frequency ratio of 9/7 (.36257 Octave). ~~Compare with 6×((1/π+1)/3)~~. This Large Fourth superimposed six times and subtracted from 3 Octaves is .36338 Octave. The nested 2-Interval-patterns are (1), 2, 3, 5, 7, 9, 16, 25, 41, 66, 91, 157, 223, 289, and breaking at 355 (5×71). This is really a very lovely tuning. There is "golden" sequence ~~from~~ thru 7, 9, 16, 25, 41, 66 which makes for excellent melody. 66, (a smaller breakpoint) it should be noted is a multiple of 22 (2×3×11).

Item 7: $(1/\pi + 1)/6 = .219718314$ Octave. (This tuning is aligned to Item 6, bisecting its Large Fourth.) Six of these Small minor Thirds superimposed give 1 Octave plus the Major Third ($1/\pi$). Chains of small minor Thirds very much like this would seem to have very much influenced the way Pelog is actually tuned. (I hasten to add Pelog is poorly understood.) This is a very Pacific sound. Nested 2-interval-patterns occur at (1), 2, 3, 4, 5, 9, 14, 23, 32, 41, 50, 91, 132, 223, and with again a major breakpoint at 355 (5×71). A golden sequence occurs at 4, 5, 9, 14, 23 which is prime melodic range. It is interesting that Jap Kunst considered 23 to be a measure of Pelog. Also ref Hornbostle's cycle of 23* (overblown Fifths).

Item 8: $(1/\pi + 1)/7 = .188329983$ Octave. This large Second superimposed 7 times gives the Octave plus $1/\pi$ Third. I have seen a tuning quite similar to this on the Andean Queña producing a Pentatonic/Hexatonic scale by large Seconds. This is related to S'lendro of Java and may well be within S'lendro tolerance. The nested 2-interval patterns occur at (1), 2, 3, 4, 5, 6, 11, 16, 21, 37, 53, 69, 85, 154, 223, 377, 600, 977, 1354, 1731, 2108, and 2485 (5×7×71) which is at the big breakpoint.

Item 9: $(1/\pi + 2)/8 = .289788735$ Octave. This tuning is aligned to item 3 whose Fifth it divides in half. The result is the "neutral" Third found through-out world music, almost anywhere but in Western Europe. (This neutral Third corresponds very closely to the frequency ratio $11/9$ (.289506617 Octave) which occurs commonly in aliquot flutes.) This neutral Third superimposed 8 times gives 2 Octaves and the $1/\pi$ Third. The nested 2-interval-patterns occur at (1), 2, 3, 4, 7, 10, 17, 24, 31, 38, 69, 107, 176, 245, 421, 666, 1087, 1753, and ~~2844~~ 2840 ($2^3 5 \cdot 71$) which is the breakpoint.

Item 10: $(5 - 1/\pi)/16$ ~~$(1 - 1/\pi + 4)/16$~~ $= .292605632$ Octave. This tuning is tied to item 1, the Fifth of which it divides in 2 parts, producing again a neutral Third ~~but more~~ similar to the frequency ratio $27/22$ and $11/9$ about equally. This neutral Third superimposed 16 times gives 5 Octaves less the $1/\pi$ Third. In the 17-tone 2-interval-pattern $1/\pi$ will be the "closing interval" in the cycle of otherwise neutral Thirds. Altho it occors only once, it occurs nevertheless at the most definitive position in the scale. It's an intriguing relationship. The nested 2-interval-patterns occur at (1), 2, 3, 4, 7, 10, 17, 24, 41, 65, 106, 147, 188, 229, 270, 311, 352, 663, 974, 1285, 1596, 1907, 2218, 2529, and 2840 ($2^3 \times 5 \times 71$) where, as in item 9, it makes a major break. This tuning has an excellent Turkish/Arabic/Persian quality about it. It also lends itself to a remarkably decent bagpipe scale.

Scale Tree, (0/1 thru 1/1)

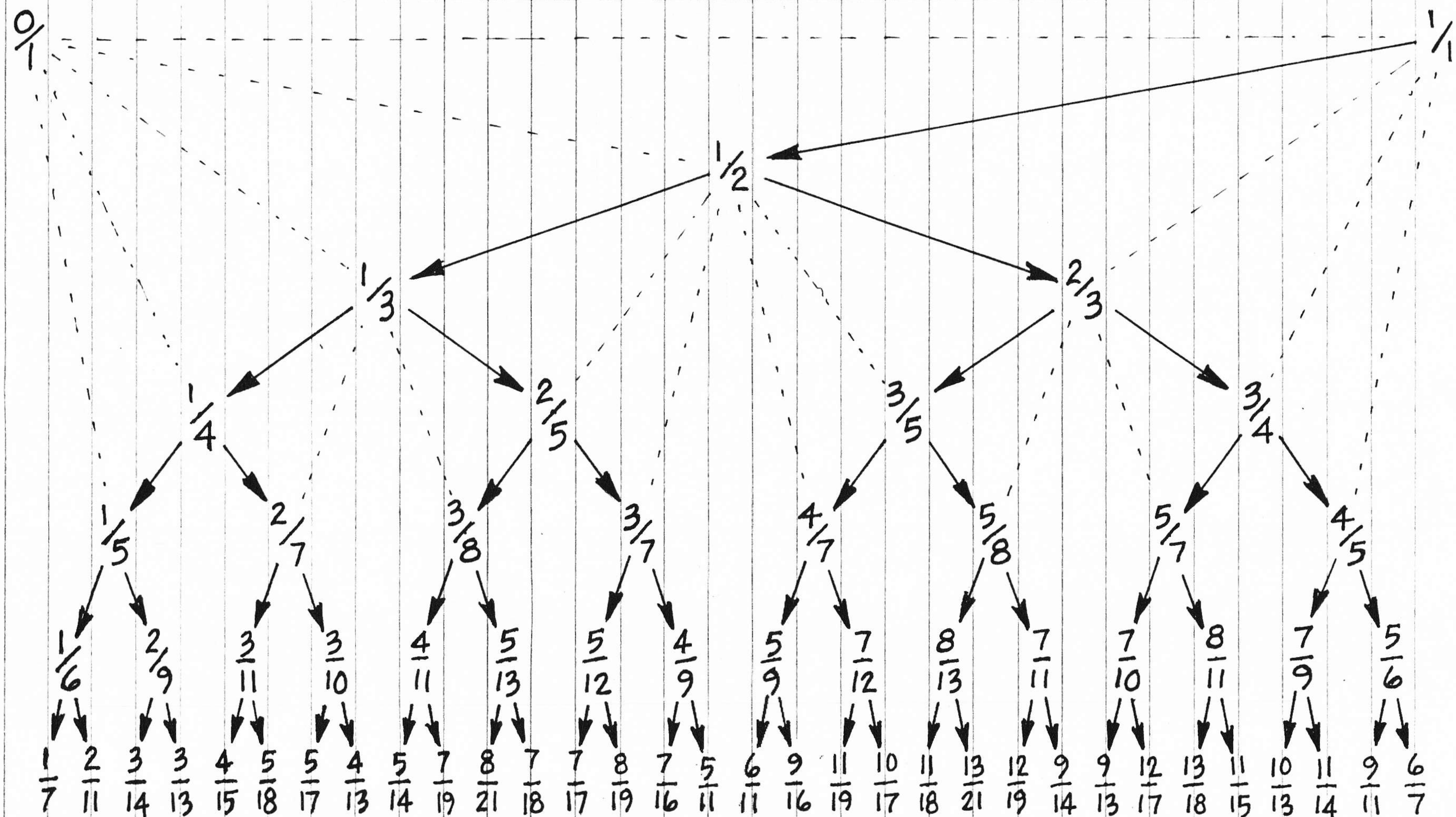


Figure 2

Note; the r.h. half of this branch is the Octave complement of the l.h. half. It is included nonetheless for reader convenience, as when the Fifth, not the Fourth, is used as the generator.

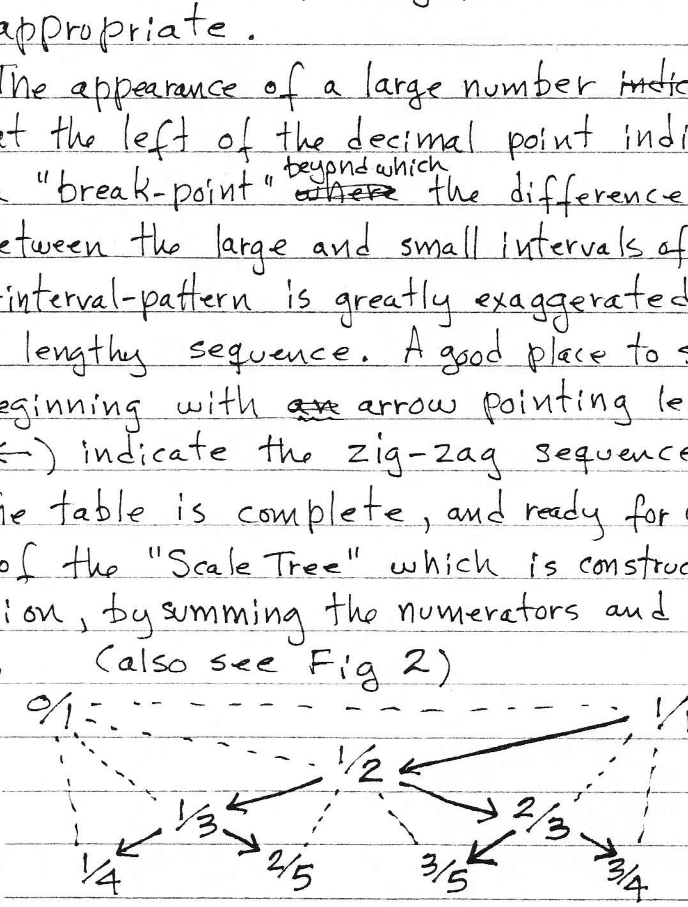
The "generator" or "generating interval" is that interval which projected about the cycle of the Octave, produces ~~the~~ a sequence of 2-interval-patterns. These 2-interval-patterns may be identified by a fraction where the numerator is the number of intervals ~~in~~ in the generator, and the denominator is the number of intervals in the Octave. It is possible to predict the sequence of 2-interval-patterns from the generator when it is expressed as a decimal fraction of the Octave, and where the Octave is taken as 1.000; First create a 1/x zig-zag table — by inverting the generator

		.579577471 ←	(example, Harrison's Fifth)
←	1	.725394877 ←	Subtract the number left of the decimal
→	1	.378559502 ←	Point and tabulate it in a column to the left.
←	2	.641 etc ←	Invert (1/x) the remaining decimal value,
→	1	.558 etc	repeating the same operation as above.
←	1	.790 ---	Continue this repeating process as
→	1	.265 ---	appropriate.
←	3	.764	The appearance of a large number ~~indicates~~
→	1	.307	at the left of the decimal point indicates
←	3	.249	a "break-point" ~~where~~ beyond which the difference
→	4	.013	between the large and small intervals of the
←	74	.188 ←	2-interval-pattern is greatly exaggerated, for
↑			a lengthy sequence. A good place to stop.

Beginning with ~~an~~ arrow pointing left (←) indicate the zig-zag sequence

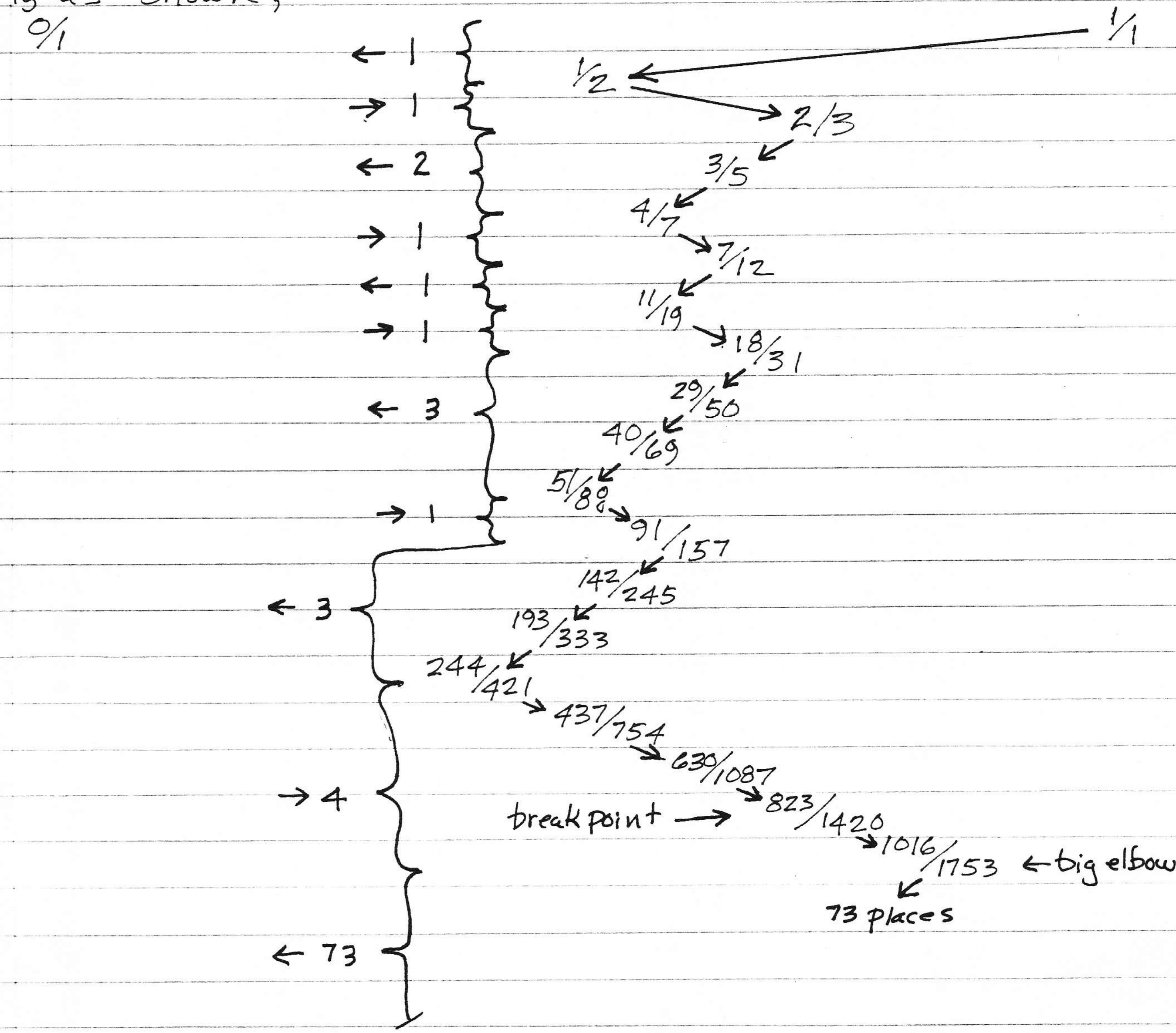
The table is complete, and ready for use.

The next operation is in the context of the "Scale Tree" which is constructed by a process of iterated triangulation, by summing the numerators and denominators respectively as shown: (also see Fig 2)



etc.

Each layer of the tree will have 1 member of the nested 2-interval-pattern series associated to any given generator. The nested sequence will be connected by a (usually) irregularly zig-zagging single branch (connected by the arrows in Figure 2) start with 1/1 in the upper right of tree. Refer to the table of numbers from the left of the decimal point: ← means zig to the left as many moves as indicated by the corresponding number, and → means zag to the right corresponding to the appropiate number. The pattern is as shown;



Item 11: $(1/\pi + 5)/13 = .40910076$ Octave. (This is the complementary tuning to Item 2.) This Fourth superimposed 13 times gives 5 Octaves and the $1/\pi$ Third. The nested 2-interval-patterns are ~~(1)~~, at (1), 2, 3, 5, 7, 12, 17, and at <u>22</u> (2×11) which is a significant break-point. 22 times this Fourth exceeds 9 Octaves by .00021673 Octave, which interval is impractical to fret. A further breakpoint occurs much further down the series at 4615 (5×13×71).

Item work sheets are enclosed.

I've got to sort my seeds for planting of Quinoa and relatives in about 2 week in northwest Chihuahua. — Hope these will give you some productive cues on the varied resources of π as a tonal determinant. I should very much like to hear more about your work with the Dolphins.

Yours Sincerely,
Erv Wilson

$(1/\pi + 3)/8 = .414788735$

(10)

$\boxed{\text{ITEM 1}}$

work sheet

$1/\pi$ zig-zag pattern

.414788735

←	2	.410
→	2	.433
←	2	.304
→	3	.281
←	3	.555
→	1	.800
←	1	.249
→	4	.006
←	146	.603

0/1 - - - - - - 1/1

1/2

1/3

2/5

3/7

5/12

7/17

12/29

17/41

22/53

39/94

56/135

73/176

129/311

202/487

331/798

460/1109

589/1420

20×71 → .414788732

718/1731

.414789139

$(1/\pi + 5)/9 = .59092332$

$1/\pi$ Zig-Zag Pattern

←	1	.692
→	1	.444
←	2	.249
→	4	.006
←	144	.969
→	1	.031
←	31	.621
→	1	.608
←	1	.642
→	1	.557

$\boxed{\text{ITEM 2}}$

work sheet

0/1 - - - - 1/1

1/2

2/3

3/5

4/7

7/12

10/17

13/22

→ .59090909

16/27

29/49

etc to

.590923416 1875/3173 #143

$(3^2 \cdot 5 \cdot 71)$ → .590923317 1888/3195 #144

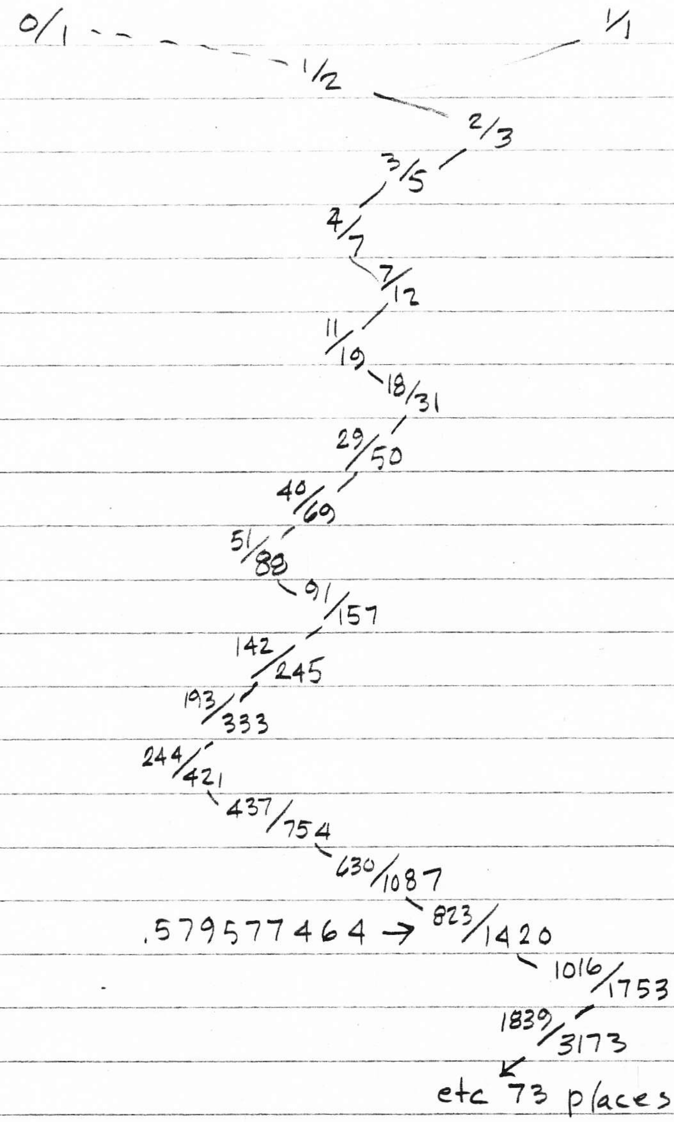
$(1/\pi + 2)/4 = .579577471$

ITEM 3

work sheet

$1/\pi$ Zig-Zag Pattern

←	1	.725	
→	1	.378	
←	2	.641	
→	1	.558	
←	1	.790	
→	1	.265	
←	3	.764	
→	1	.307	
←	3	.249	
→	4	.013	
←	73	.188	spur
	5	.298	
	3	.345	
	2	.893	
	1	.119	
	8	.396	
	2	.520	(4×5×71)
	1	.920	
	1	.086	
	11	.621	
	1	.609	
	1	.640	
	1	.561	
	1	.782	
	1	.278	
	3	.588	
	1	.699960324	
	1	.428652404	
	2	.332892548	
	3	.003972317	
	251	.7422204 !	another spur
	1	.347	



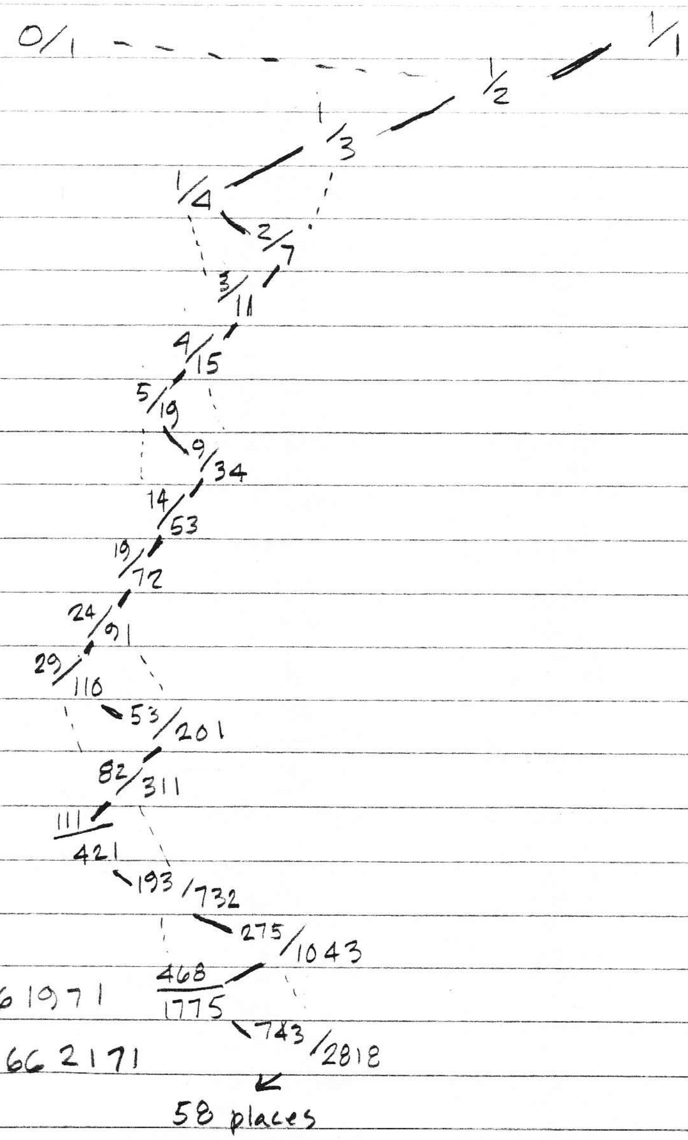
$(1/\pi + 1)/5 = .263661977$

ITEM 4

worksheet

$1/\pi$ Pattern

←	3	.792
→	1	.261
←	3	.824
→	1	.212
←	4	.705
→	1	.416
←	2	.399
→	2	.504
←	1	.983
→	1	.017
←	58	.129



71 X 25 → .2636619771

.2636621711

ITEM 5

worksheet

$1/\pi = .318309886$

$1/\pi$ zig-zag pattern

←	3	.141
→	7	.062
←	15	.996
→	1	.003
←	292 spur	.634
	1	etc
	1	
	1	
	2	cont
	1	5
	4	4
	1	1
	2	9
	14	2
	16	2
	13	2
	1	8
	11	3
	1	1
	1	14
	3	2
	1	129 another spur
	1	

0/1 1/1

1/2

→ .333333333 1/3

1/4

2/7

3/10

4/13

5/16

6/19

→ .318181818 7/22

8/25

15/47

22/69

29/91

36/113

43/135

50/157

57/179

64/201

71/223

78/245

85/267

92/289

99/311

106/333

(5×71) → .318309859 113/355

219/688

332/1043

etc to

.318309886	33102/103993	# 291
.318309886	33215/104348	# 292

$(1/\pi + 1)/3 = .439436628$

ITEM 6

work sheet

<u>$1/\pi$ zig-zag Pattern</u>

←	2	.275
→	3	.627
←	1	.592
→	1	.687
←	1	.454
→	2	.200
←	4	.998
→	1	.001
←	880	.012

0/1   1/1

1/2

1/3

2/5

3/7

4/9

7/16

11/25

18/41

29/66

40/91

69/157

98/223

127/289

→ (5x71) .439436619   156/355

283/644

439/999

etc 880 places !

$(1/\pi + 1)/6 = .219718314$

ITEM 7

work sheet

$1/\pi$ zig-zag pattern

←	4	.551
→	1	.813
←	1	.228
→	4	.375
←	2	.666
→	1	.500
←	1	.999
→	1	.000567
←	1761	.234

0/1 - - - - - - - - - 1/1

1/2

1/3

1/4

1/5

2/9

3/14

5/23

7/32

9/41

11/50

20/91

29/132

49/223

→ (5×71) .219718309 78/355

127/578

205/933

etc 1761 places!

137485/625733 off the keyboard

$(1/\pi + 1)/7 = .188329983$

ITEM 8

work sheet

$1/x$ zig-zag pattern

←	5	.309
→	3	.227
←	4	.393
→	2	.538
←	1	.856
→	1	.167
←	5	.976
→	1	.024
←	41	.097

→ (35×71) .188329979

0/1

1/1
1/2
1/3
1/4
1/5
1/6
2/11
3/16
4/21
7/37
10/53
13/69
16/85
29/154
42/223
71/377
113/600
184/977
255/1354
326/1731
397/2108
468/2485

$(1/\pi + 2)/8 = .289788735$

ITEM 9

work sheet

$1/x$ Zig-zag Pattern

←	3	.450
→	2	.218
←	4	.580
→	1	.723
←	1	.382
→	2	.615
←	1	.624
→	1	.601
←	1	.663
→	1	.506
←	1	.973
→	1	.027
	36	.083

0/1 1/1
1/2
1/3
1/4
2/7
3/10
5/17
7/24
9/31
11/38
20/69
31/107
51/176
71/245
122/421
193/666
315/1087
508/1753
(40×71) .289788732 ⟶ 823/2840
1331/4593
etc 36 places

$(1-(1/\pi))+4)/16 = .292605632$

ITEM 10

work sheet

1/x Zig-Zag Pattern

3	.417
2	.394
2	.532
1	.876
1	.140
7	.110
9	.031
73	.298

0/1 . . . 1/1

1/2

1/3

1/4

2/7

3/10

5/17

7/24

.292682926 12/41

19/65

31/106

43/147

55/188

67/229

79/270

.292604501 91/311

103/352

194/663

285/974

376/1285

467/1596

558/1907

649/2218

740/2529

(40×71) .292605633 → 831/2840

922/3151

→ etc 73 places

$(1/\pi + 5)/13 = .40910076$

ITEM 11

worksheet

$1/x$ zig-zag pattern

	.40910076
2	.444
2	.250
3	.995
1	.004
208	.955

$1/0 \ldots$ $1/1 \to 1/2 \to 1/3 \to 2/5 \to 3/7 \to 5/12 \to 7/17 \to 9/22$ (2×11) $\to 16/39 \to 25/61$ etc 208 places

$1879/4593$ #207

$1888/4615$ #208

.409100758 ! (5×13×71)

Fractional Commatic Meantone Temperaments, Scale-Tree Sort

ref. Tuning and Temperament by J. Murray Barber 1953

 20 July 2003. EW

John Harrison's $\frac{\left(\frac{1}{\pi}\right)+2}{4} = .5795774715459\ldots$

$\frac{0}{1}$		
$\frac{1}{8}$		
$\frac{1}{7}$	79	
$\frac{2}{13}$	14	
$\frac{1}{6}$	67	Silbermen
$\frac{3}{17}$	189	
$\frac{2}{11}$	122	
$\frac{3}{21}$	177	
$\frac{1}{5}$	55	Verheijen
$\frac{4}{19}$	153	
$\frac{3}{14}$	98	
$\frac{5}{23}$	141	
$\frac{2}{9}$	43	Rossi
$\frac{5}{22}$	117	
$\frac{3}{13}$	74	
$\frac{4}{17}$	105	
$\frac{1}{4}$	31	Zarlino, Vincentino
$\frac{5}{19}$	143	
$\frac{4}{15}$	112	
$\frac{7}{26}$	193	
$\frac{3}{11}$	81	Romieu
$\frac{8}{29}$	212	Kornerup about here
$\frac{5}{18}$	131	
$\frac{7}{25}$	181	
$\frac{2}{7}$	56	Zarlino
$\frac{7}{24}$	169	
$\frac{5}{17}$	119	
$\frac{8}{27}$	188	
$\frac{3}{10}$	69	Romieu
$\frac{7}{23}$	157	
$\frac{4}{13}$	88	Harrison about here
$\frac{5}{16}$	107	
$\frac{1}{3}$	19	Salinas

1/2 comma meantone Fifth

.576002

18 Jul 2003 · ew

	.576002	
←	1	.736
→	1	.358
←	2	.789
→	1	.266
←	3	.748
→	1	.336
←	2	.975
	1	.024
	40	.660

0/1 . . . 1/1

1/2, 2/3, 3/5, 4/7, 7/12, 11/19, 15/26, 19/33, 34/59, 53/92, 72/125, 125/217, 197/342

40 pl.

epimoria
2/1
4/3
10/9
21/20
49/48
133/132
286/285
495/494
1,122/1,121
3,128/3,127
6,625/6,624
15,625/15,624
42,750/42,749

Golden section

	1.618034	
→	1	.618
←	1	.618
→	1	.618
←	1	.618
←	1	.618
→	1	.618
←	1	.618
	etc	

1/1 . . . 1/0

2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55, 144/89, 233/144, 377/233, 610/377, 987/610, 1,597/987, 2584/1,597

	epimoria
1/1	2/1
2/1	4/3
3/2	10/9
5/3	25/24
8/5	65/64
	169/168
	442/441
	1,156/1,155
	3,026/3,025
	7,921/7,920
	20,737/20,736
	54,289/54,288
	142,130/142,129
	372,100/372,099
	974,170/974,169
	2,550,409/2,550,408

π Zig-Zag

2OCT00.EW

π = 3.141592653...

1/x Pattern	
3	.141
7	.062
15	.996
1	.003
292	.634
1	.575
1	.738
1	.354

Zig-Zag Pattern

1/1 → 2/1 → 3/1 → 4/1   1/0

7/2 10/3 13/4 16/5 19/6 22/7 ← does this occur in the literature?

25/8 → 47/15 69/22 91/29 113/36 135/43 157/50 179/57 201/64

– 3.14

223/71 245/78 267/85 289/92 311/99 333/106 355/113 ← !!

688/219 → 279 places